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Tournament tables, Power-Weakness Ratio and Hasse diagrams: an informative combination for multi-criteria decision-making.

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Prologue

The starting points of this work are our two previous papers published in 2015:

R. Todeschini, F. Grisoni, S. Nembri, (2015)
Weighted power-weakness ratio for multi-criteria decision making.
Chemometrics and Intelligent Laboratory Systems, 146, 329-336.

F. Grisoni, V. Consonni, S. Nembri, R. Todeschini (2015)
How to weight Hasse diagrams and reduce incomparabilities.
Chemometrics and Intelligent Laboratory Systems, 147, 95-104.

Tournament table

The results of a Round Robin tournament of N players can be conveniently expressed by mean of a tournament table (dominance matrix) as:

	P1	P2	P3	...	...	PN
P1	0	t_{12}	t_{13}	...	...	t_{1N}
P2	t_{21}	0	t_{23}	...	...	t_{2N}
P3	t_{31}	t_{32}	0	...	...	t_{3N}
...	...	...	...	...	...	
...	...	...	...	...	...	...
PN	t_{N1}	t_{N2}	t_{N3}	...	...	0

$$t_{ij} + t_{ji} = 1$$

H.A. David (1971)
Ranking the Players in a Round Robin Tournament.
Review of the International Statistical Institute, 39, 137-147.

Tournament table

The results of a Round Robin tournament of N players can be conveniently expressed by mean of a tournament table (dominance matrix) as:

	P1	P2	P3	...	...	PN
P1	0	**1**	t_{13}	...	...	t_{1N}
P2	**0**	0	t_{23}	...	...	t_{2N}
P3	t_{31}	t_{32}	0	...	...	t_{3N}
...	...	...	...	...	...	
...	...	...	...	...	...	...
PN	t_{N1}	t_{N2}	t_{N3}	...	...	0

$$t_{ij} + t_{ji} = 1$$

H.A. David (1971)
Ranking the Players in a Round Robin Tournament.
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Tournament table

Tournament table $\mathbf{T_1}$. For each t_{ij}: 1 if the P_i player won over P_j, 0 if P_j won, 0.5 if they drew the match.

$\mathbf{T_1}$	P1	P2	P3	P4	P5
P1	0	1	1	0.5	0
P2	0	0	1	0.5	0.5
P3	0	0	0	1	1
P4	0.5	0.5	0	0	0.5
P5	1	0.5	0	0.5	0

$$t_{ij} + t_{ji} = 1$$

Sometimes, some conflicting rings arise:

1) Ranking cannot be decided
2) Transitivity property is lost

$$P1 > P3 > P5 > P1$$

The row sum (Copeland score) can used for ranking:

P1: 2.5

P2 = P3 = P5 = 2

P4 = 1.5

… but the ranking power can be low!

Tournament table

A tournament table can be derived from any data matrix **X** (N,p), where N is the number of objects and *p* the number of variables, i.e. the considered criteria.

$$\mathbf{X} \rightarrow \mathbf{T_W}$$

The general expression for this transform is defined by comparing objects pairwise:

$$t_{ij}^{W} = \sum_{k=1}^{p} w_k \cdot \delta_{ij,k} \quad \text{where} \quad \delta_{ij,k} = \begin{cases} 1 & \text{if } x_{ik} \rhd x_{jk} \\ 0.5 & \text{if } x_{ik} \triangleq x_{jk} \\ 0 & \text{if } x_{ik} \lhd x_{jk} \end{cases} \quad \text{and} \quad \sum_{k=1}^{p} w_k = 1$$

… where the main differences with respect to the Hasse approach are …

Tournament table

A tournament matrix can be derived from any data matrix **X** (N,p), where N is the number of objects and *p* the number of variables, i.e. the considered criteria.

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A set of thresholds are also derived from the tournament table:

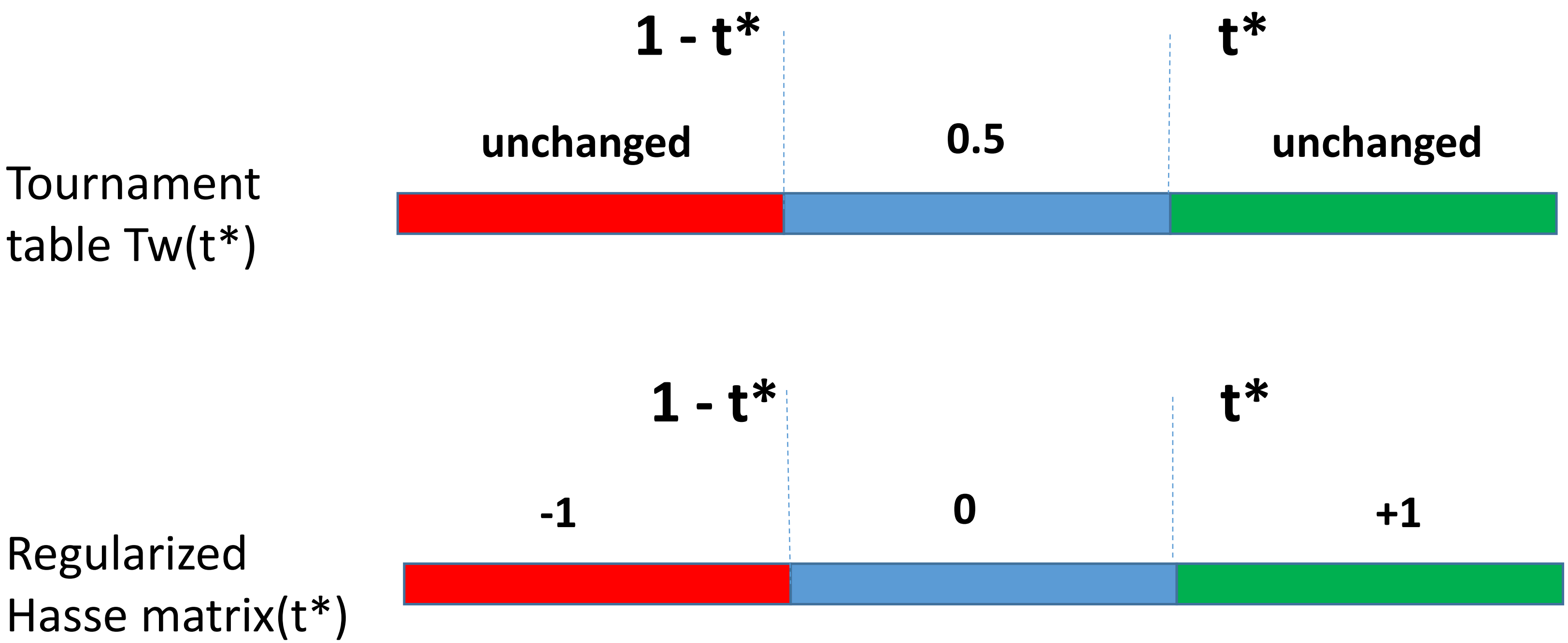
$$\mathbf{X} \rightarrow \mathbf{T_W} \rightarrow \{t_1, t_2, \ldots, t_k\}$$

Tournament table

Analyzing thresholds of the tournament table

$$0.5 \leq t^* \leq 1$$

The following **transforms** are performed



Power-Weakness Ratio

For any squared asymmetrical matrix, the **Perron-Frobenius theorem** guarantees the existence of a positive eigenvalue associated with an eigenvector **e** having positive values.

Tournament table Tw

Kendall (1955) proposed to use the **eigenvector values** to rank the objects, thus also removing possible lost of transitivity:

$$\mathbf{T_W} \rightarrow \mathbf{e}$$

Ramanujacharyulu (1964) proposed to use also the eigenvector values calculated on the transpose of **Tw**:

$$\mathbf{T}_W^T \rightarrow \mathbf{e}^*$$

Power-Weakness Ratio

… then the ***PWR*** of the *i*-th object was defined as:

$$PWR_i = \frac{e_i}{e_i^*}$$

Indeed, the first eigenvector awards good players able to win with other good players, while the second eigenvector characterizes bad players which loss with other bad players.

Power-Weakness Ratio

Tournament table $\mathbf{T_1}$. For each t_{ij}: 1 if the P_i player won over P_j, 0 if P_j won, 0.5 if they drew the match.

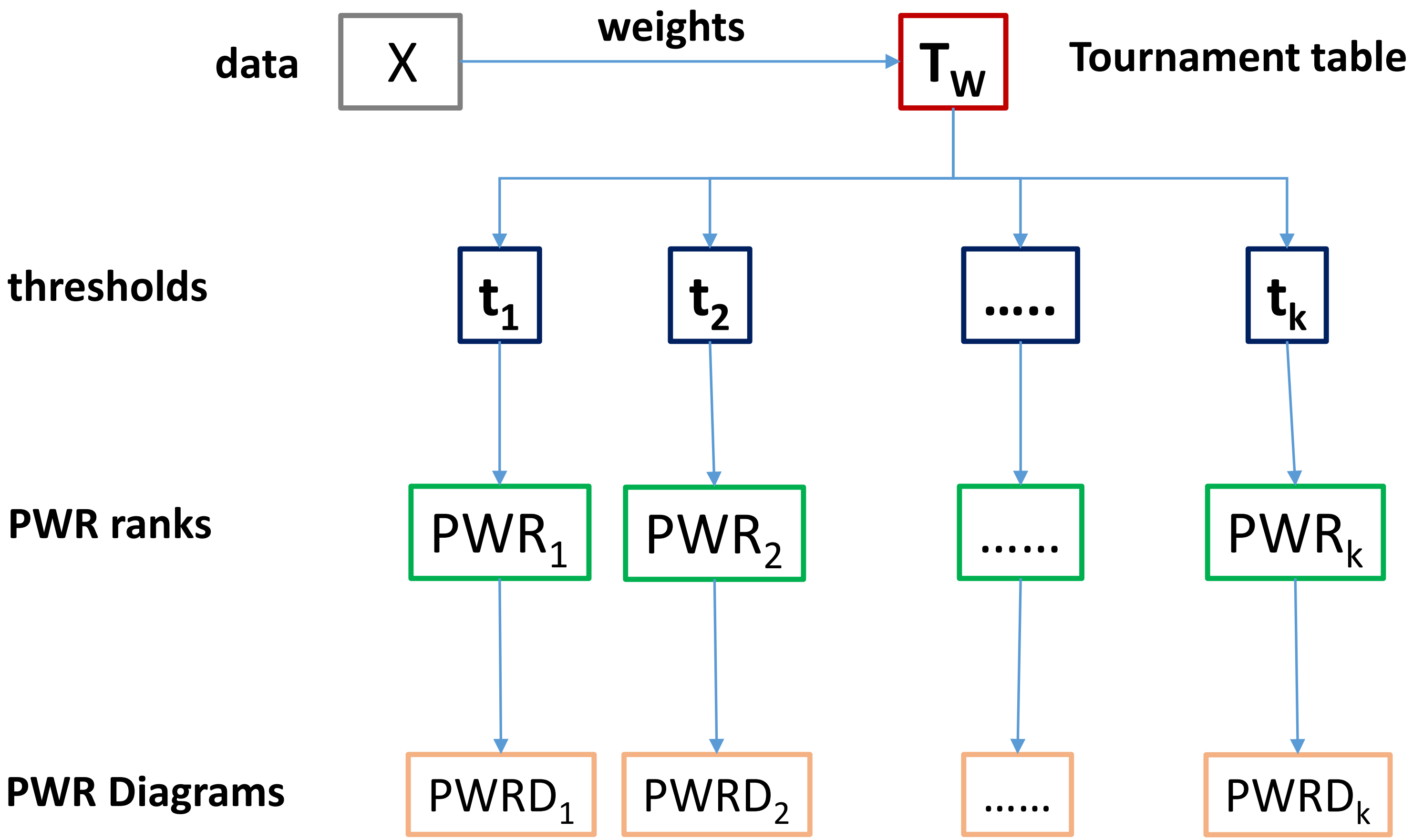
$\mathbf{T_1}$	P1	P2	P3	P4	P5
P1	0	1	1	0.5	0
P2	0	0	1	0.5	0.5
P3	0	0	0	1	1
P4	0.5	0.5	0	0	0.5
P5	1	0.5	0	0.5	0

Results of PWR scoring on table $\mathbf{T_1}$. Entries of the Perron–Frobenius eigenvector calculated on tournament table ($\mathbf{e}_{\mathrm{PF}}$) and on its transpose ($\mathbf{e}^*_{\mathrm{PF}}$) for each player are also reported.

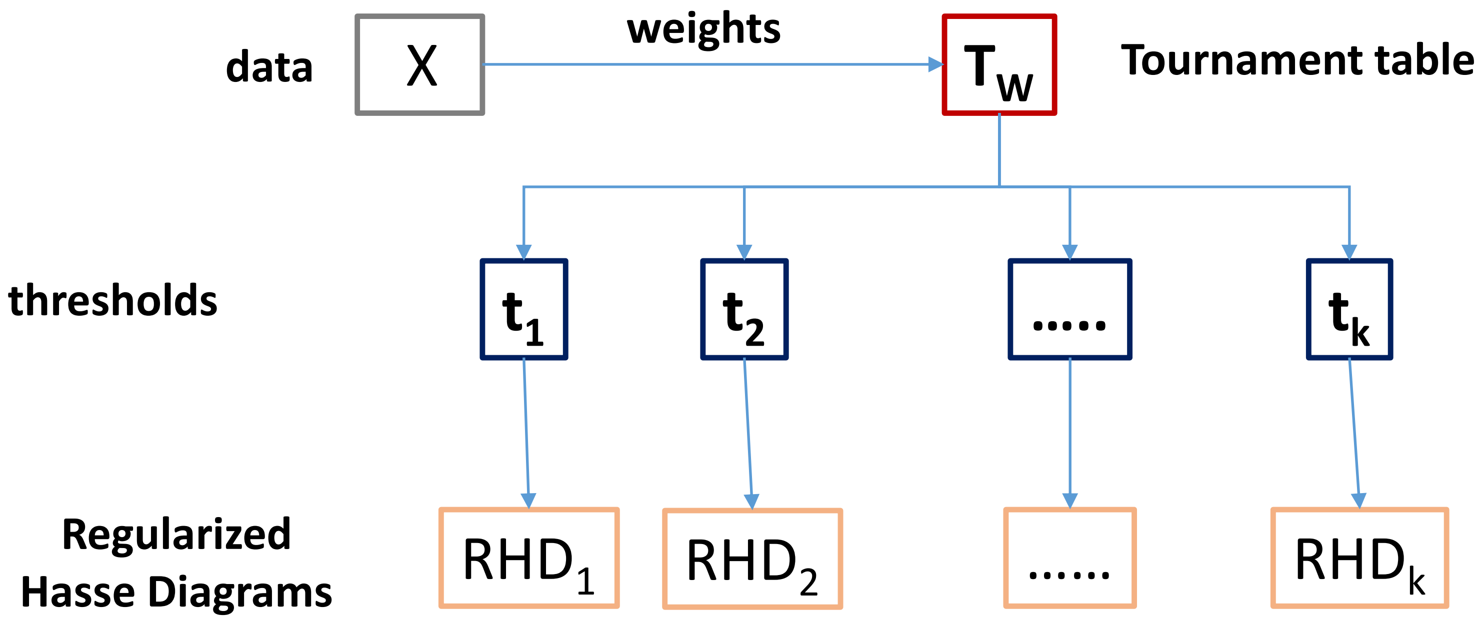
Players	e_{PF}	$e_{\mathrm{PF}}{}^*$	PWR	RS
P1	0.529	0.370	1.368	2.5
P2	0.430	0.442	0.976	2.0
P3	0.426	0.414	1.025	2.0
P4	0.364	0.535	0.714	1.5
P5	0.471	0.459	1.023	2.0

Eigenvector (T_W)   Eigenvector (T^T_W)

Tw transform

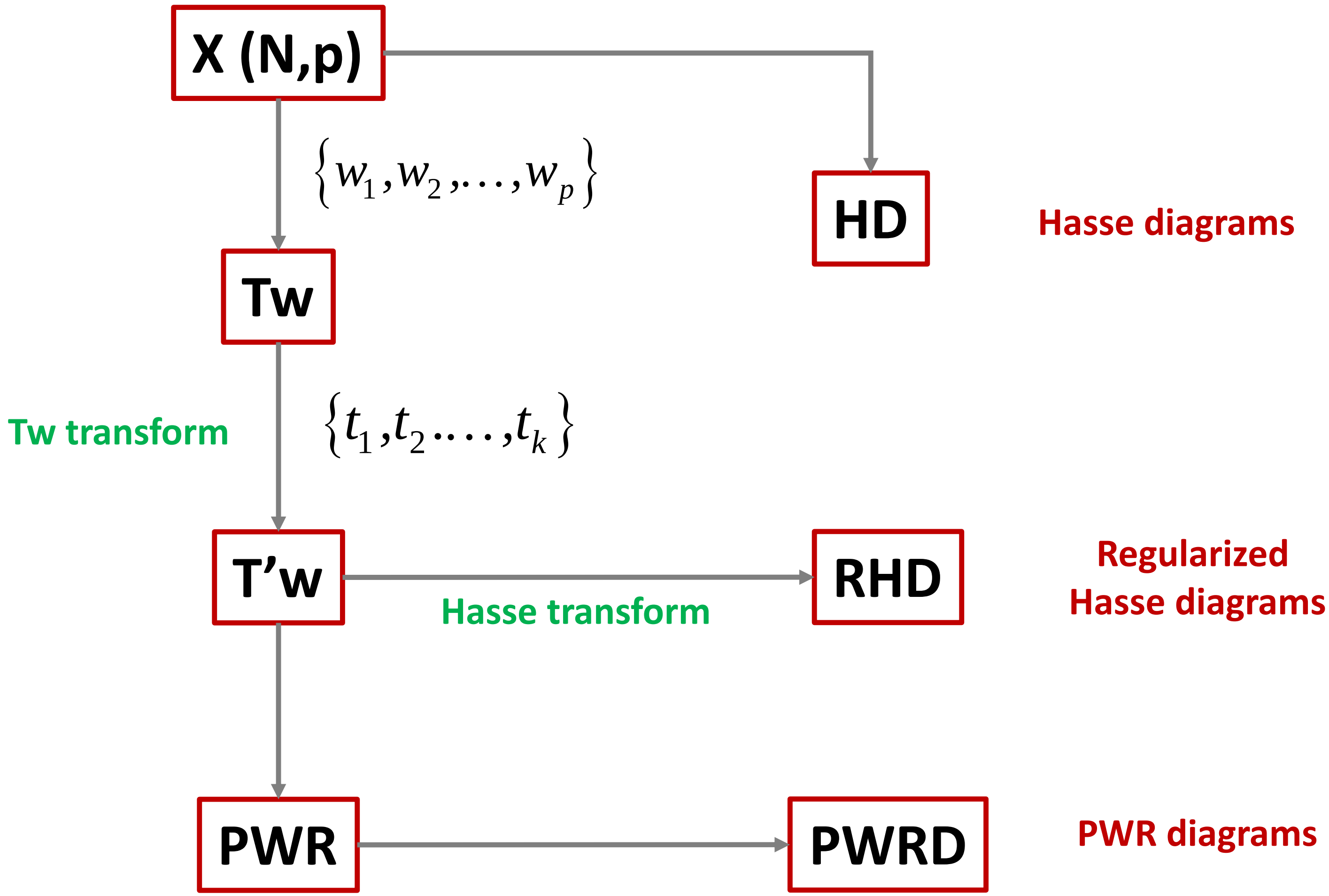


Hasse transform



$$\left[\mathbf{H}^{\mathrm{R}}\left(t^{*}\right)\right]_{ij} = \begin{cases} +1 & if\ t_{ij}^{\mathrm{W}} \ge t^{*} \\ -1 & if\ t_{ij}^{\mathrm{W}} \le 1-t^{*} \\ 0 & otherwise \end{cases} \qquad 0.50 < t^{*} \le 1$$

Summary



X (N,p)
$\{w_1, w_2, \ldots, w_p\}$
HD
Hasse diagrams
Tw
Tw transform
$\{t_1, t_2 \ldots, t_k\}$
T'w
RHD
Hasse transform
Regularized Hasse diagrams
PWR
PWRD
PWR diagrams

Comparisons of classification methods

- **32 data sets**
- **Validation procedure: leave-one-out**
- **Parameter: Non-Error-Rate (NER%)**

10 CLASSIFIERS

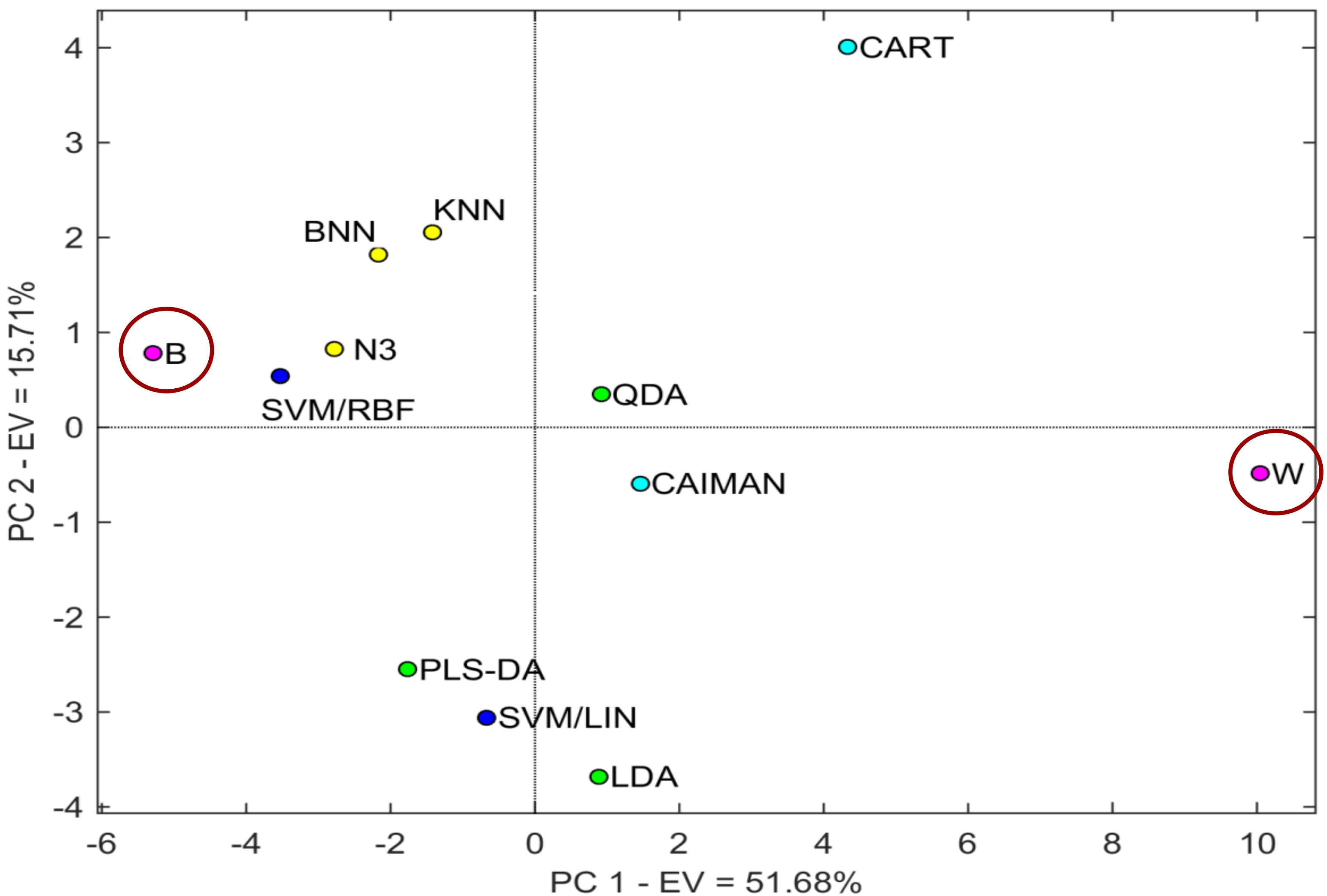
N3	**PLS-DA**
BNN	**CAIMAN**
KNN	**CART**
LDA	**SVM/LIN**
QDA	**SVM/RBF**

Comparisons of classification methods

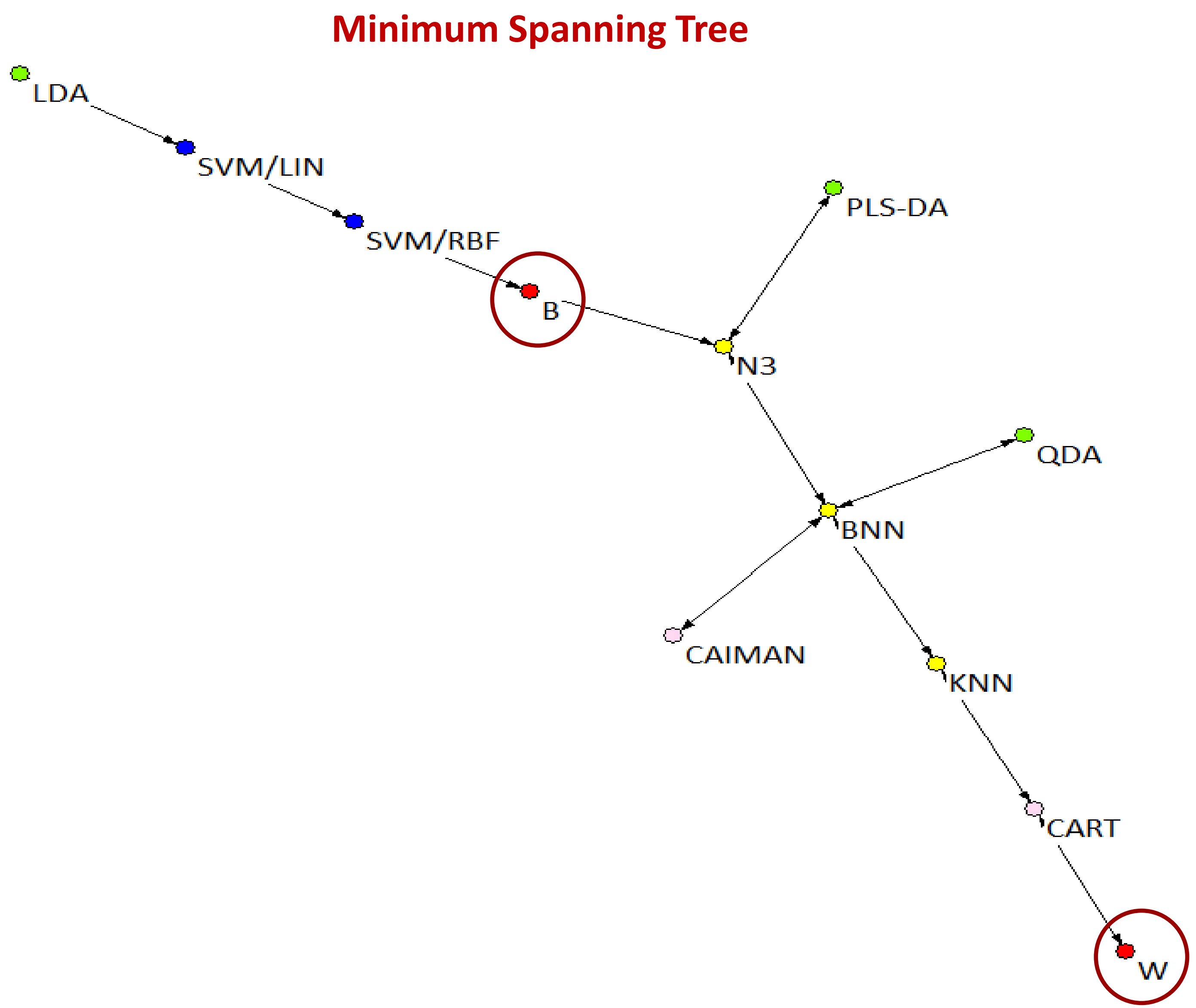
Id	Data set	N3	BNN	KNN	LDA	QDA	PLSDA	CART	CAIMAN	SVM /LIN	SVM /RBF
1	IRIS	96.0	96.7	96.7	**98.0**	97.3	90.2	94.0	**98.0**	97.3	97.3
2	WINES	96.2	98.6	97.7	99.1	**99.5**	**99.5**	86.2	98.7	99.1	**99.5**
3	PERPOT	99.0	99.0	99.0	85.0	92.0	86.0	97.0	97.0	87.0	**100.0**
4	ITAOILS	**96.2**	95.2	94.7	94.7	95.9	95.9	87.2	82.8	94.7	95.9
5	SULFA	77.4	73.8	73.8	45.2	69.4	74.0	81.5	58.7	50.0	**88.7**
6	DIABETES	73.6	71.1	70.5	72.7	69.6	**75.1**	68.8	73.5	72.3	72.8
7	BLOOD	67.9	62.2	62.3	53.7	54.5	**68.7**	62.1	59.3	50.0	64.1
8	VERTEBRAL	80.8	81.6	80.2	80.7	84.0	82.1	76.9	56.0	83.3	**84.3**
9	SEDIMENTS	88.9	88.9	**89.9**	66.9	69.4	79.4	84.3	61.1	50.5	69.9
10	BIODEG	84.5	85.3	**85.4**	77.0	78.6	79.9	79.6	65.6	81.5	83.8
11	DIGITS	74.2	72.3	73.6	74.0	68.6	41.0	65.2	**77.3**	74.9	74.5
12	APPLE	94.0	92.3	91.9	91.9	87.6	**95.4**	92.1	83.9	94.4	92.3
13	TOBACCO	92.3	92.3	92.3	84.6	80.8	88.5	**96.2**	92.3	92.3	92.3
14	SCHOOL	95.3	**96.6**	96.2	90.8	95.2	89.4	86.8	95.0	94.0	96.4
15	BANK	86.9	**91.2**	86.9	86.5	88.5	84.9	86.5	88.5	88.5	88.9
16	HIRSUTISM	88.3	90.1	90.0	55.4	81.4	84.1	70.5	52.9	72.7	**93.8**
17	THIOPHENE	83.3	83.3	83.3	79.2	79.2	**90.5**	58.3	83.3	83.3	83.3
18	SUNFLOWERS	92.3	90.4	91.2	87.8	90.8	92.7	82.1	88.9	90.8	**96.9**
19	VINAGRES	**100.0**	91.7	95.8	**100.0**	87.5	**100.0**	67.3	**100.0**	**100.0**	**100.0**
20	CHEESE	76.1	78.3	78.1	78.8	82.9	84.7	63.9	77.5	76.2	**85.6**
21	ORUJOS	98.2	**98.4**	98.2	92.6	94.1	93.9	88.4	62.5	95.7	98.2
22	MEMBRANE	94.4	94.4	94.4	88.9	94.4	**96.7**	91.7	94.4	91.7	94.4
23	METHACYCLINE	82.5	**86.7**	82.5	45.8	81.7	55.8	65.8	80.0	54.2	82.5
24	SIMUL4	**100.0**	**100.0**	**100.0**	28.1	**100.0**	46.9	90.6	93.8	34.4	**100.0**
25	VEGOIL	99.0	**100.0**	99.0	98.0	82.2	99.0	99.3	89.9	99.3	**100.0**
26	CRUDEOIL	89.2	84.8	87.9	85.2	73.6	**89.7**	64.9	78.4	85.3	84.8
27	SAND	93.9	**94.9**	93.9	93.9	93.9	93.9	81.9	93.9	**94.9**	**94.9**
28	HEMOPHILIA	85.6	85.6	82.8	85.6	83.9	85.6	78.9	**86.7**	**86.7**	85.6
29	COFFEE	**100.0**	**100.0**	**100.0**	**100.0**	92.9	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**
30	OLITOS	89.1	73.6	70.4	83.1	80.0	**94.0**	58.0	77.2	87.6	87.6
31	FISH	92.6	92.9	92.9	96.4	85.2	**100.0**	88.7	89.0	**100.0**	**100.0**
32	HEARTHDISEASE	**69.9**	65.2	63.2	68.8	66.2	69.7	66.1	67.3	68.0	68.0

Comparisons of classification methods

Principal Component Analysis

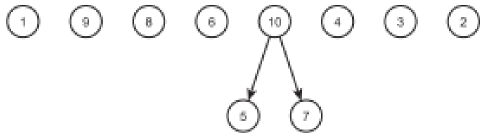


Comparisons of classification methods

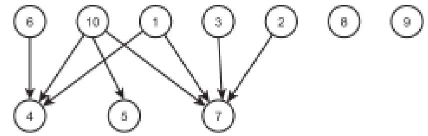


Regularized Hasse diagrams

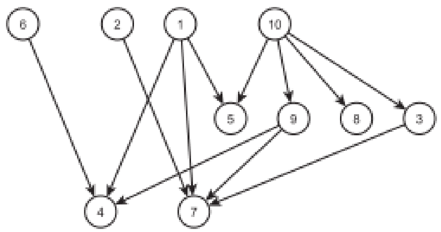
t*= 0.92



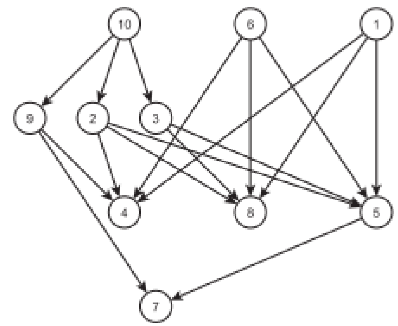
t*= 0.81



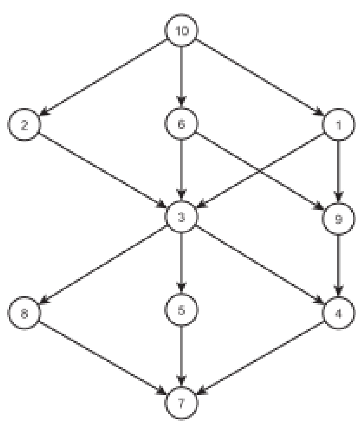
t*= 0.73



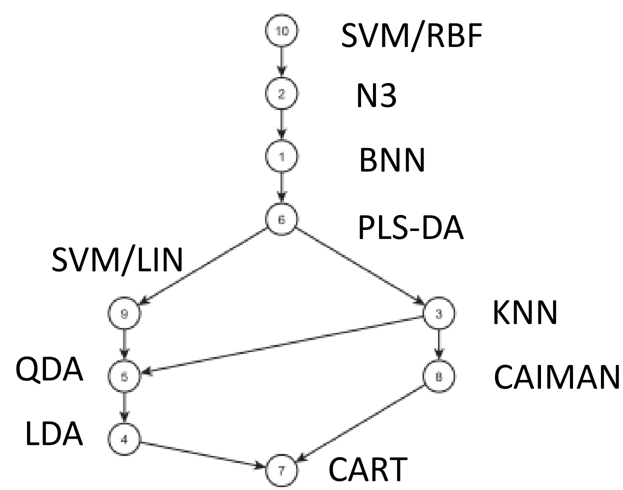
t*= 0.64



t*= 0.58

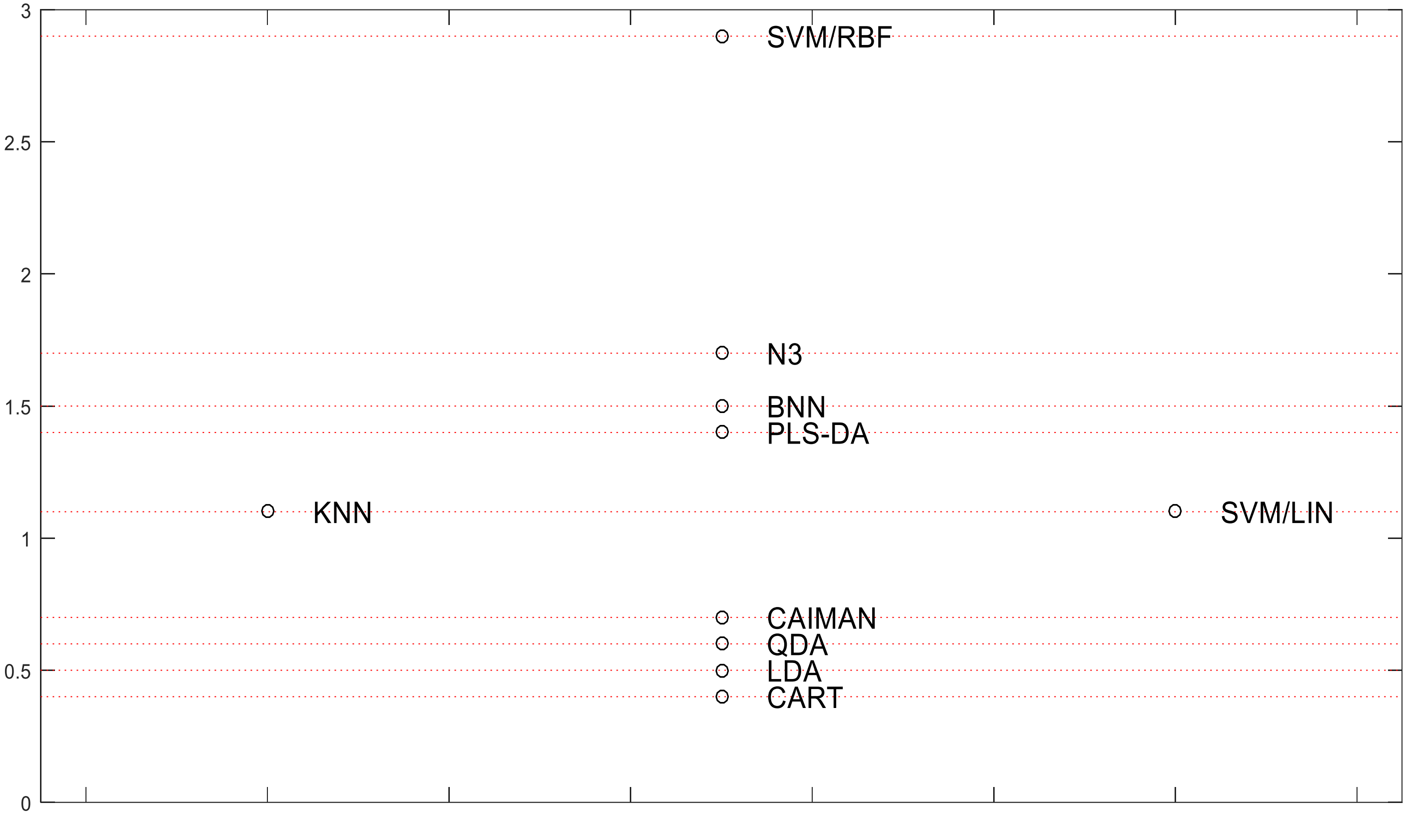


t*= 0.55

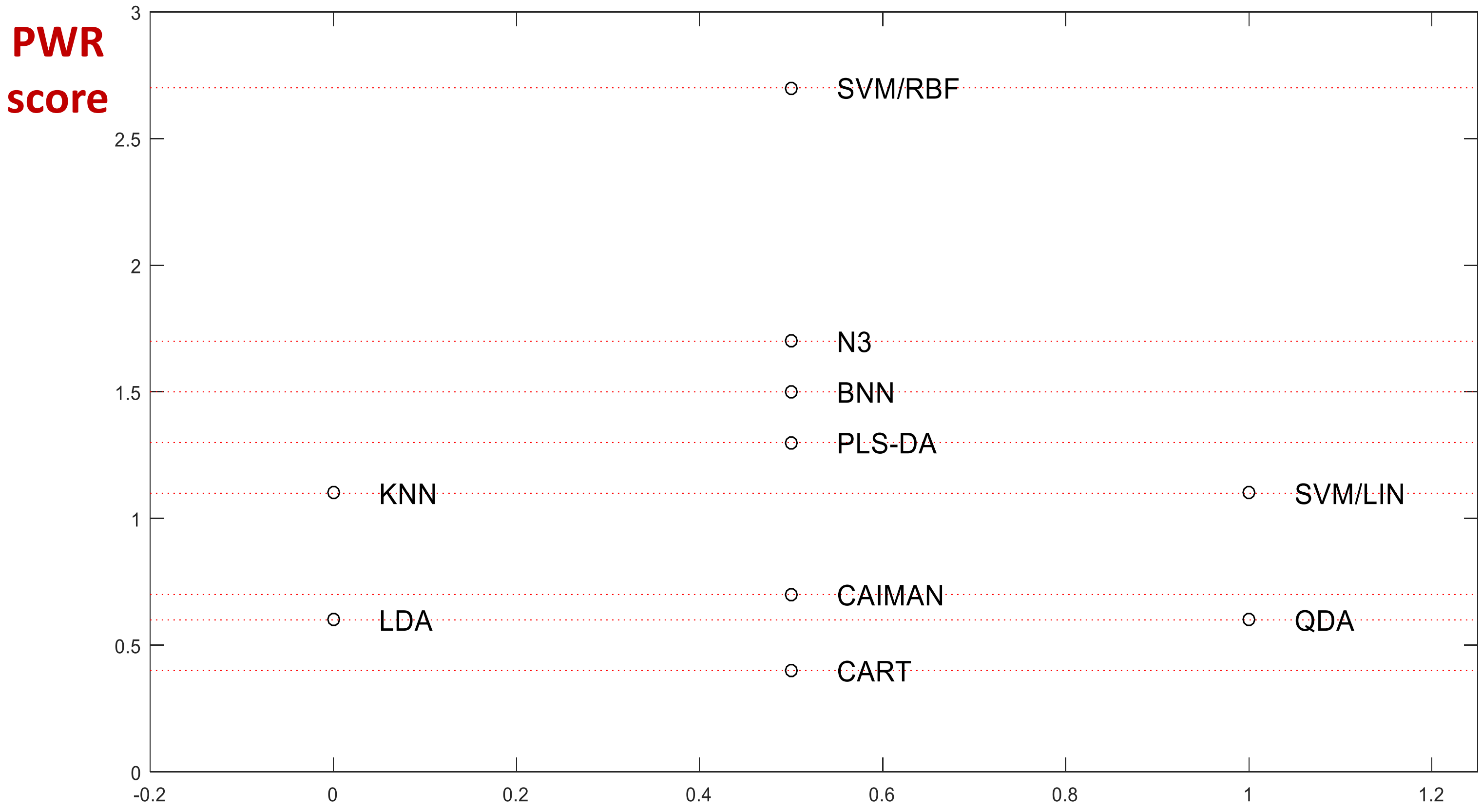


PWR diagrams (t* = 0.5)

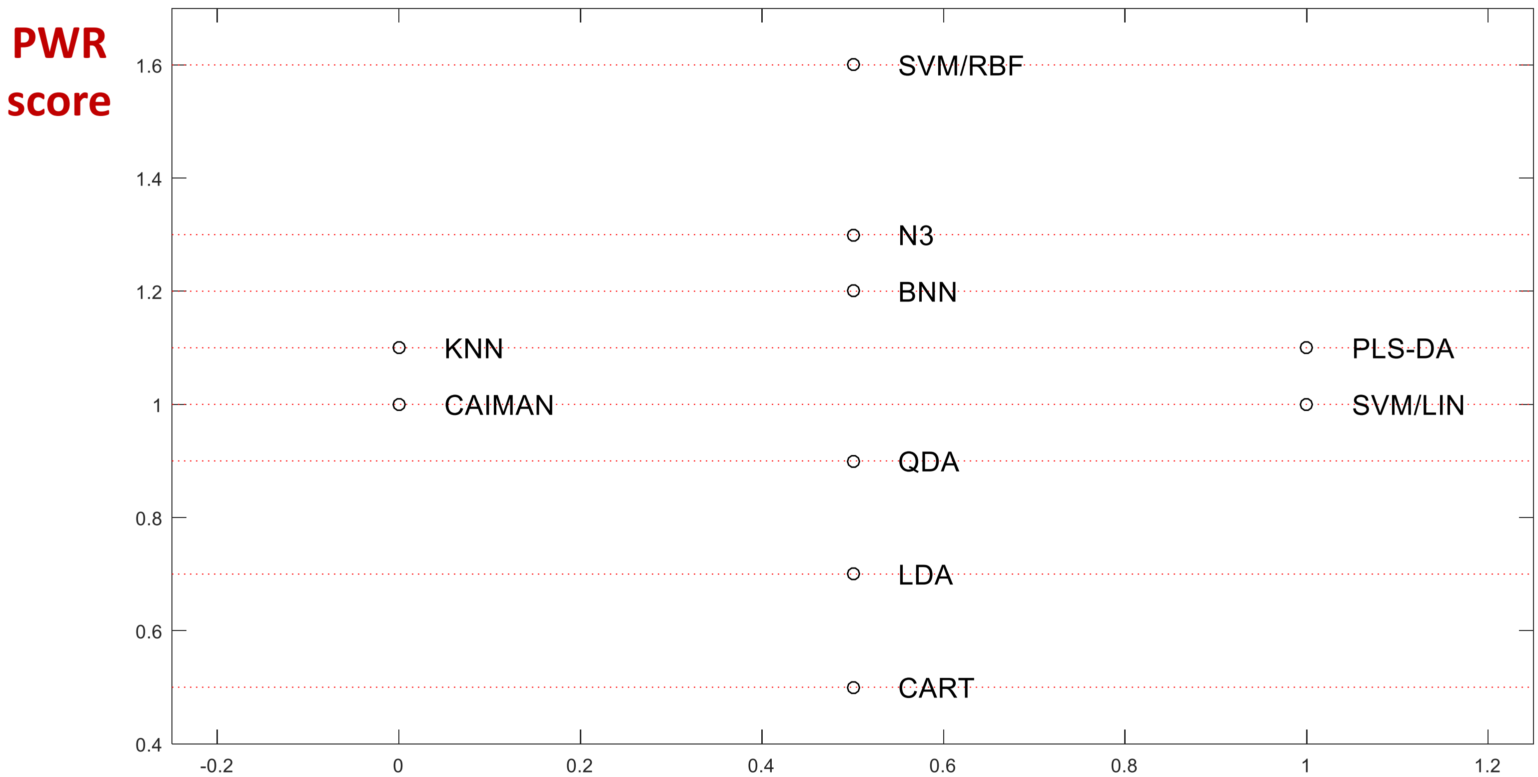
PWR score



PWR diagrams (t* = 0.6)



PWR diagrams (t* = 0.8)



Anilines data set

45 anilines described by 4 criteria:

1. log Kow (octanole-water partition coeff.)
2. log VP (vapor pressure)
3. Biodegradability (1: yes; 2: no)
4. PNEC (Predicted No-Effect Concentration)

Study focused on:

1. Hasse diagram (HD)
2. From HD to MonteCarlo ranking
3. From HD to Average ranking

L. Carlsen (2006), A combined QSAR and partial order ranking approach to risk assessment, *SAR and QSAR in Environmental Research*, 17, 133-146.

Anilines Hasse diagrams

Adverse effect

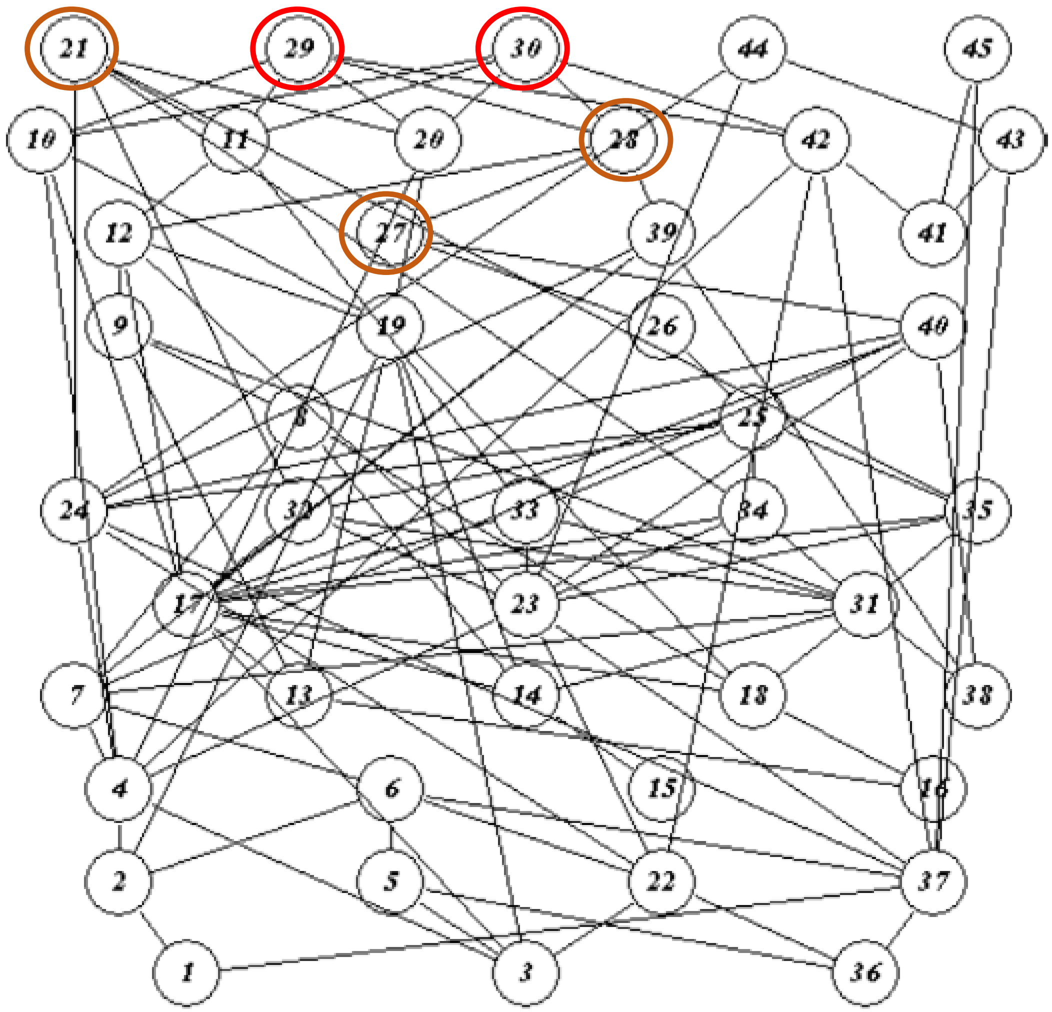
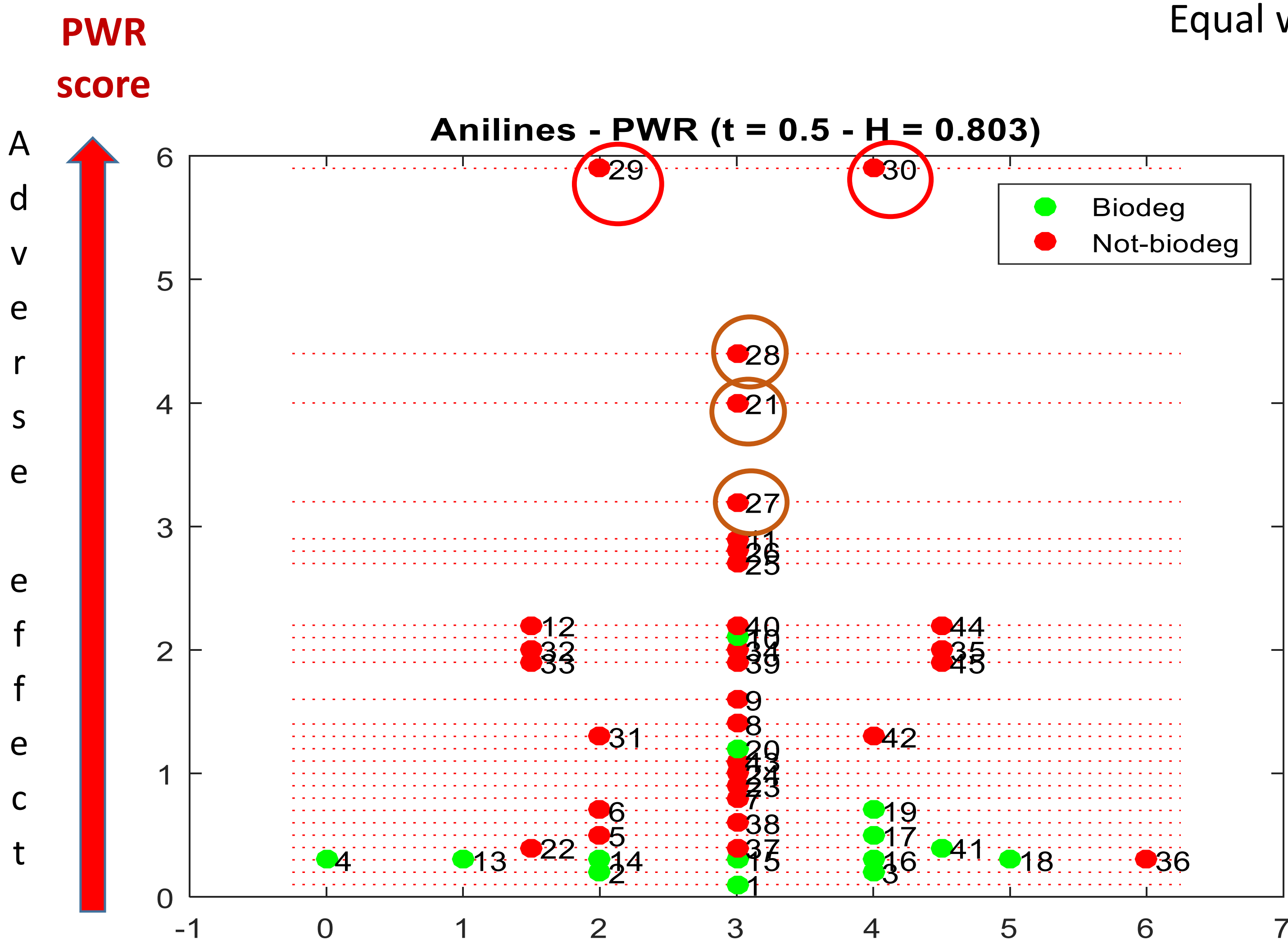


Figure 3. Hasse diagram of the 45 anilines based on the 4 descriptors given in table 2. The single compounds are identified through their ID (cf. table 2).

Anilines: PWR diagram (t* = 0.5)

Equal weights

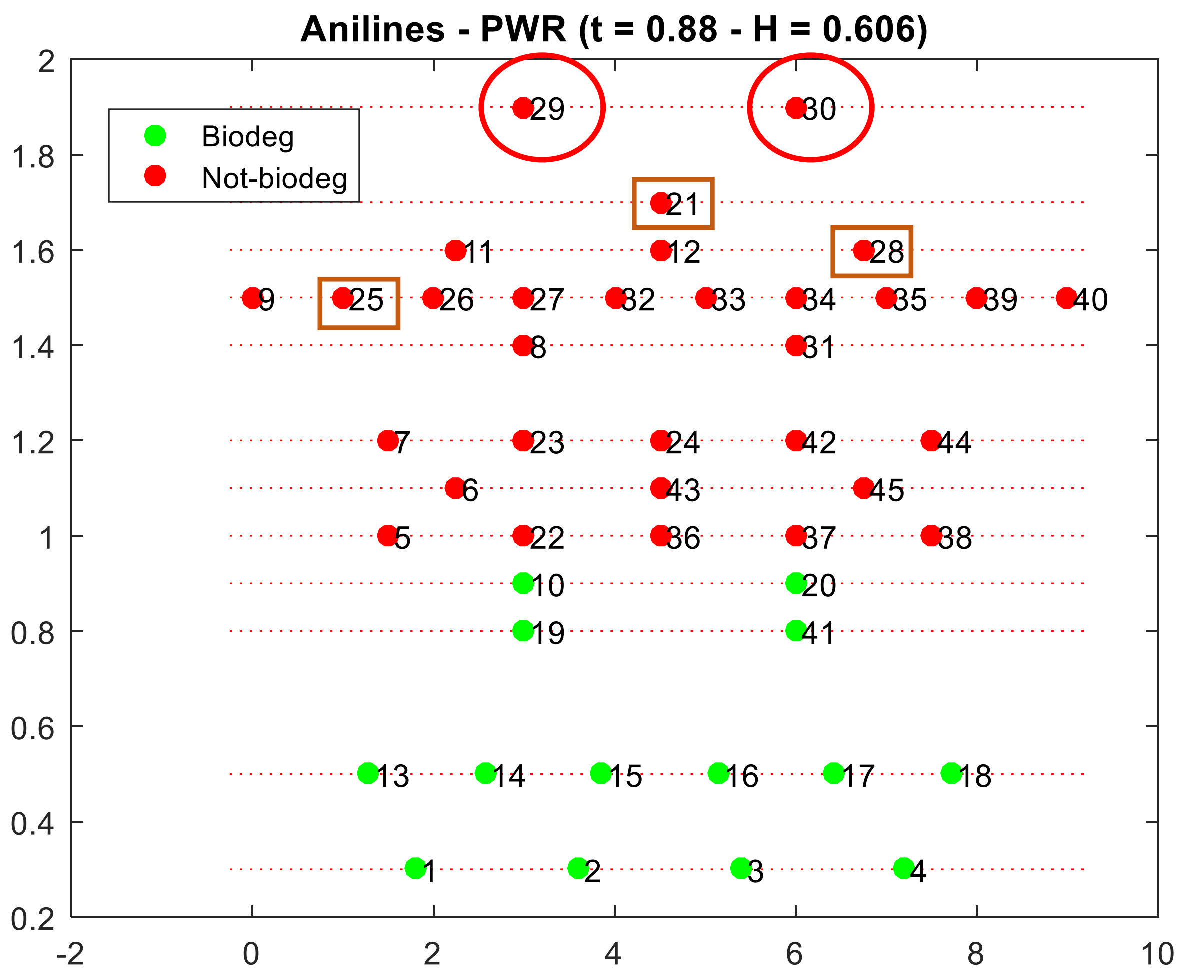


Anilines: PWR diagram (t* = 0.88)

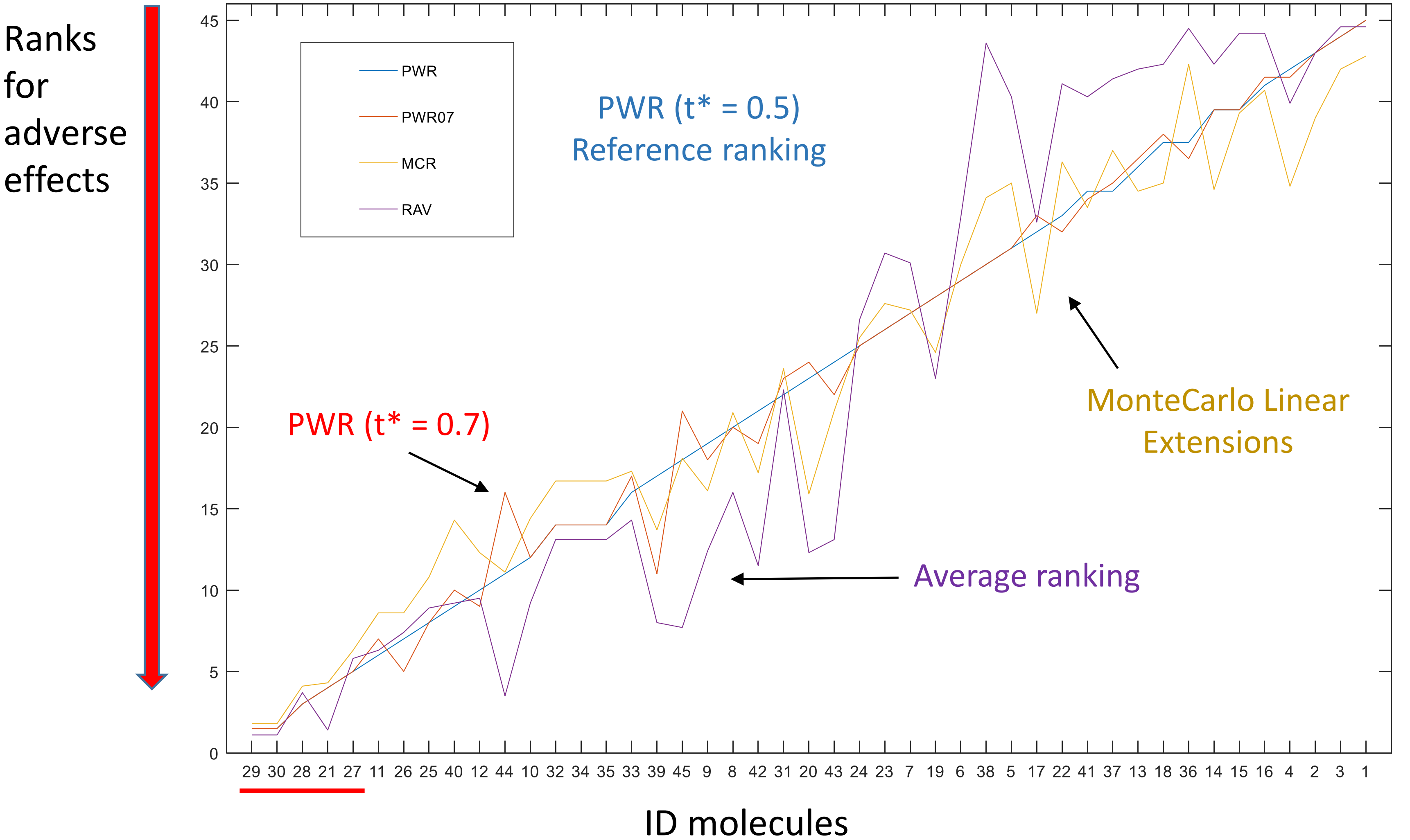
Equal weights

PWR score

Adverse effect



Anilines: ranks comparison



Conclusions

- Possibility to weight the criteria
- Threshold selection offers different opportunities to rank the objects
- The Hasse transform from tournament table produces a family of regularized Hasse diagrams, thus also allowing a reduction of incomparabilities
- PWR is able to rank objects by a well founded theory
- PWR can remove inconsistencies from the tournament table
- PWR diagrams introduce a quantitative axis
- PWR diagrams can recover several incomparabilities present in the Hasse diagrams
- Statistical analysis can be performed on both the family of regularized-Hasse diagrams and the set of PWR rankings