Introduction of weights in poset-based synthetic indicators

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Outline

1. Introduction
   - Finding a Synthetic Measure
   - Poset Theory
2. Tools from Poset theory
   - Approximation of Average rank
3. Our Proposal
   - Weights in posets
4. Case Study - Disability
   - Concept and Data
   - Results

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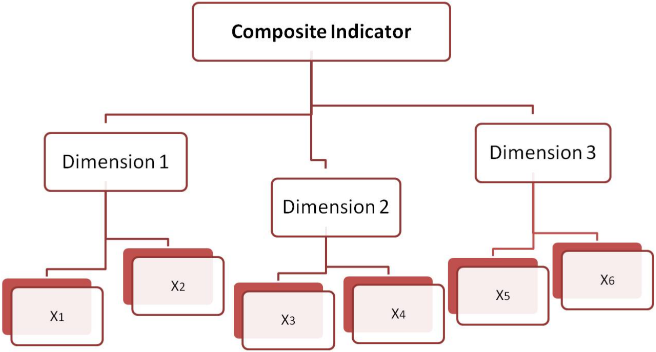
A Composite Indicator

Quality of life, Environment, Gender Equality, Human Development . . .



Quantitative elementary indicators aggregated through different methods to describe a complex and not observable concept

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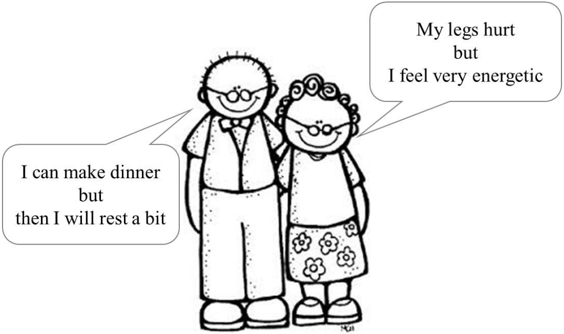
Ordinal Data

The measure of complex and unobservable concepts is the main reason of this work



We focus on micro level data, often measured on **ordinal** scale.

In social studies it implies thousands of observations.

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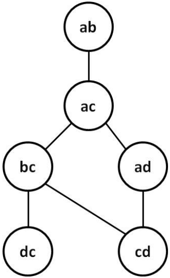
Representation - Hasse Diagram

As usual, we represent the elements of the poset with the Hasse diagram

Child	His.	Mat.	Profile
c_1	a	c	ac
c_2	c	d	cd
c_3	d	c	dc
c_4	a	d	ad
c_5	b	c	bc
c_6	a	b	ab



Six children on Maths and History

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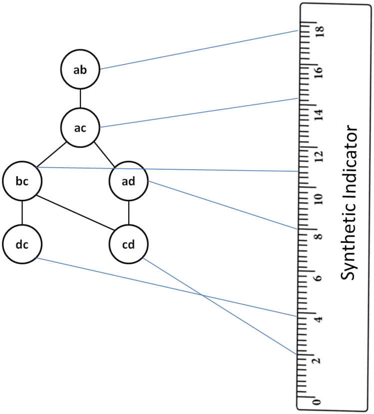
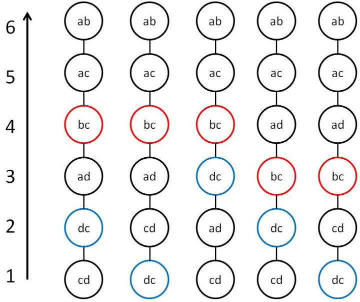
Our Aim

Starting from the order information, we look for a synthetic measure of the profiles position in the poset:



Exact Average rank

Observing the position of a profile among the linear extensions



$h(\cdot)$	ω_1	ω_2	ω_3	ω_4	ω_5	$\bar{h}(\cdot)$
ab	6	6	6	6	6	6
ac	5	5	5	5	5	5
bc	4	4	4	3	3	3.6
ad	3	3	2	4	4	3.2
dc	2	1	3	2	1	1.8
cd	1	2	1	1	2	1.4

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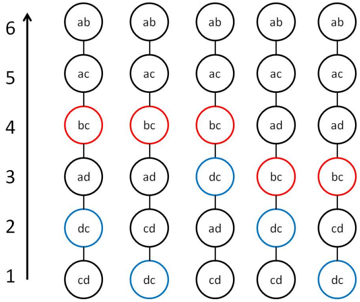

Compute the mutual rank probability of profiles $P(x \geq y)$

The average rank is
$\bar{h}(x) = \sum_y P(x \geq y)$

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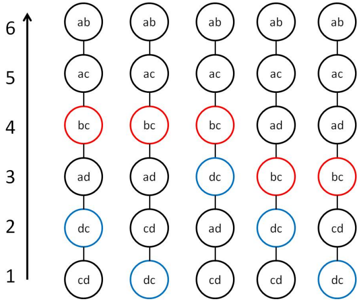

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	ab	ac	ad	bc	cd	dc	Σ	$\overline{h}(\cdot)$
ab	1	1	1	1	1	1	6	6
ac	0	1	1	1	1	1	5	5
bc	0	0	0.6	1	1	1	3.6	3.6
ad	0	0	1	0.4	1	0.8	3.2	3.2
dc	0	0	0.2	0	0.6	1	1.8	1.8
cd	0	0	0	0	1	0.4	1.4	1.4

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Computational Issues

It is not possible to define the time needed to compute the number of linear extensions in a deterministic way,

¨The Extensions are too many!¨

In order to handle with this issue, practitioners adopt two different approaches:

Sampling of linear extensions;

Approximation of average rank.

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Approximation of Average Rank

Aim to compute an approximated averaged rank $\overline{h}_a(\cdot)$ avoiding the observation of every linear extension

Definitions:

Given a Poset P, and $x \in P$,

Down set of x: $O(x) = \{y \in P : y \leq x\}$

Up set of x: $F(x) = \{y \in P : y \geq x\}$

Incomparable to x: $U(x) = \{y \in P : y||x\}$

Brüggeman and Carlsen (2011), proposed two methods based on these sets.

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Formula

The average rank is computed as

$$\bar{h}(x) = \sum_{y} P(x \geq y)$$

$$\bar{h}_a(x) = |O(x)| + \sum_{y \in U(x)} \hat{P}(x \geq y) = |O(x)| + \sum_{y \in U(x)} \eta(x, y)$$

The probability $\hat{P}(x \geq y)$ is the required information.
$\eta(x, y)$ is the quantity proposed in literature to approximate it.

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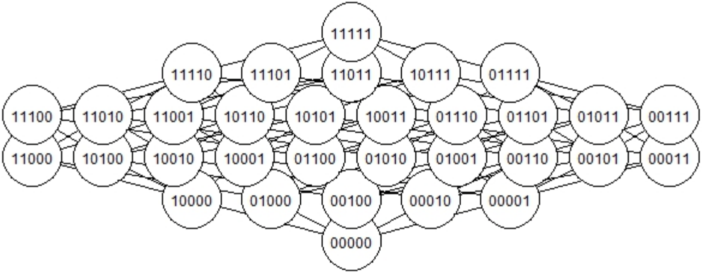
Simulated Population

The poset observing 5 dichotomous variables and no weights.



There is no difference among the nodes on the same level,
If weights are not considered, it implies *Equal Weights*.

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Position vs Evaluation

We are not only interested in the position of an individual respect to the others.

We look for an **evaluation** of its condition.
Our proposal is the introduction of weights ($\mathbf{w}$).

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$$\psi(x,y) = \sum_{i=1}^{g} [w_i \cdot I(x_i > y_i) + \frac{1}{2} w_i \cdot I(x_i = y_i)]$$

where g is the number of observed variables

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Approximation with weights

We propose to change the formula of approximation

$$\overline{h}_a(x) = |O(x)| + \sum_{y \in U(x)} \eta(x, y)$$

Including the weights of variables

$$\overline{h}_w(x, s) = |O(x)| + \sum_{y \in U(x)} \eta(x, y)^s \cdot \psi(x, y)^{1-s}$$

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for $s = 0.5$, we have the geometric mean of η and ψ.

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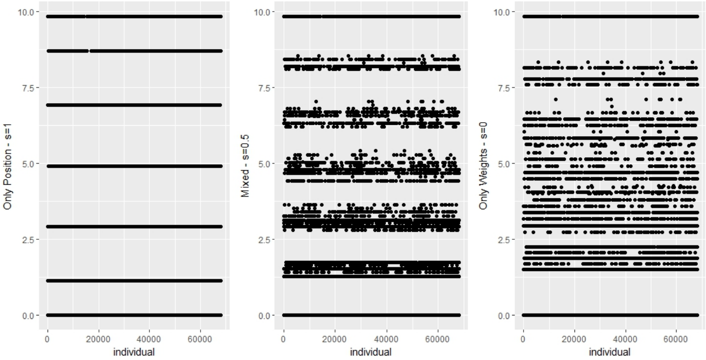
Comparison in Simulated Population

Compare $\overline{h}_a(\cdot)$, $\overline{h}_w(\cdot, 0.5)$ and $\overline{h}_w(\cdot, 0.1)$
$\mathbf{w} = \{0.05, 0.1, 0.15, 0.20, 0.25, 0.25\}$



The comparison shows relevant differences, mainly in the variability

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Definition of Disability - Katz, 1963

Disability: inability to perform the activities of daily living (ADLs)
Measure of Disability: at least one ADL is impossible for the individual

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Disability: inability to perform the activities of daily living (ADLs)
Measure of Disability: at least one ADL is impossible for the individual

ADL 1: Bathing and Showering;

ADL 2: Dressing;

ADL 3: Use of toilette;

ADL 4: Mobility;

ADL 5: Personal Care;

ADL 6: Self-Feeding;

Does every missing ADL represent the same severity of disability?

Severity of Disability

A measure of the Severity of Disability is not available.
Often the number of ADLs is considered, but this implies the same severity for every ADL.

AIM: to build a composite indicator that measures the severity of disability, keeping in mind the differences among ADLs.

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Big survey data

Data from the *Survey of Health, Ageing and Retirement in Europe* (SHARE)

More than 60 thousands of observed units in 2015 (Wave 6)

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The variables of interest are 6 items on *Activities of Daily Living*

Measured on dichotomous levels:
no - no serious limitation or **yes** - seriously limited

Weights of the ADL

The weights we used

ADL	Wash	Dress	Use Toilette	Mobility	Personal Care	Self-Feed
Weights	0,05	0,05	0,23	0,10	0,03	0,54

Elicited from expert's opinion with the AHP approach.

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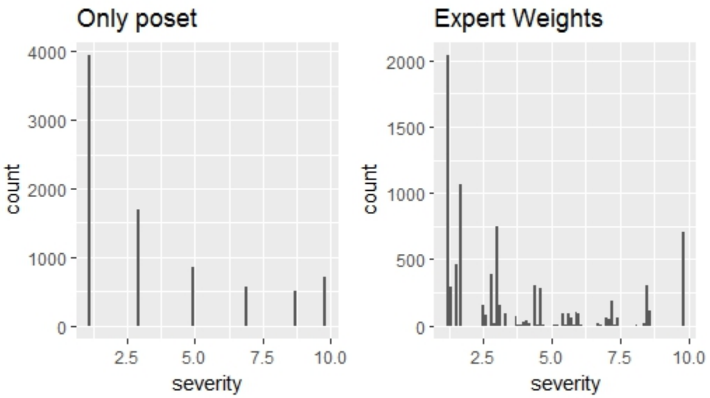
Results



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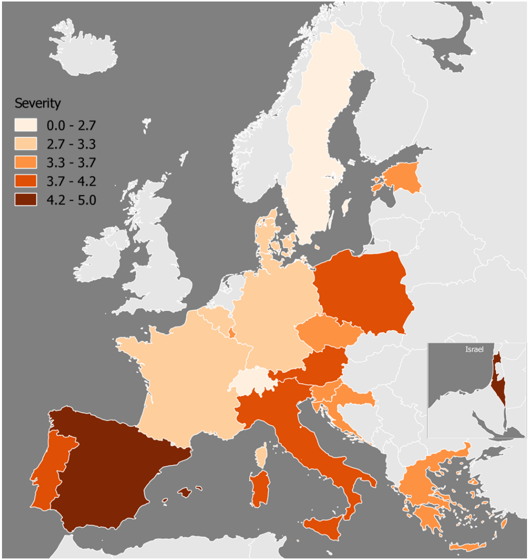
Disability in Europe



The distribution of Disability Severity is uneven across Europe, with higher values in the eastern and southern countries

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Conclusions

- This method fits to dichotomous data;
- The weights can be simply integrated and tuned;
- The results are realistic with easy use and interpretation;
- Future steps:

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- This method fits to dichotomous data;
- The weights can be simply integrated and tuned;
- The results are realistic with easy use and interpretation;
- Future steps:
  1. Extend the group of experts, to underline different perspectives;
  2. Perform sensitivity analysis on the effect of the *s*-value.

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