# Some Efficient Sampling Designs Based on Partial Order Sets

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- **Part 4:** Multivariate variable sampling based on Partial Order Theory.

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- **Part 1:** Improve sampling designs based on ranks of data (RSS).
- **Part 2:** An economic version of RSS (VSR).
- **Part 3:** Multivariate VSR.
- **Part 4:** Multivariate variable sampling based on Partial Order Theory.
- **Part 5:** Simulations.

# What are we looking for in Sampling Theory?

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- Efficient Sampling Designs


  - Low Cost
  - High Precision

# Assumptions and Notations

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- The goal is to estimate $\mu$ and the variance of the estimator.
- Suppose further that $Y$ is an auxiliary variable (used for ranking) and suppose that $Y$ has reasonable correlation with the main variable $X$.

# Improve sampling designs based on ranks of data (RSS)

# Part 1

# Simple Random Sampling with $n = 3$ (sample size), X is height.

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- Select $m$ sets of size $m$ based on SRS
- Sort each set, based on an auxiliary variables
- Full measure the main diagonal of the matrix as the final sample

| | | Min | Second Min | $\cdots$ | Max |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $\cdots$ | $X_{[m]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $\cdots$ | $X_{[m]2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m}$ | $X_{[2]m}$ | $\cdots$ | $X_{[m]m}$ |

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| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $\cdots$ | $X_{[m]2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m}$ | $X_{[2]m}$ | $\cdots$ | $X_{[m]m}$ |

# RSS of Size $n_\bullet = nm \Longrightarrow n$ times a RSS of size $m$

# RSS of Size $n_\bullet = nm \Longrightarrow n$ times a RSS of size $m$

| Cycle | Set | Ranks 1 | 2 | $\cdots$ | m |
|---|---|---|---|---|---|
| 1 | 1 | $X_{[1]11}$ | $X_{[2]11}$ | $\cdots$ | $X_{[m]11}$ |
| | 2 | $X_{[1]21}$ | $X_{[2]21}$ | $\cdots$ | $X_{[m]21}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m1}$ | $X_{[2]m1}$ | $\cdots$ | $X_{[m]m1}$ |
| 2 | 1 | $X_{[1]12}$ | $X_{[2]12}$ | $\cdots$ | $X_{[m]12}$ |
| | 2 | $X_{[1]22}$ | $X_{[2]22}$ | $\cdots$ | $X_{[m]22}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m2}$ | $X_{[2]m2}$ | $\cdots$ | $X_{[m]m2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| n | 1 | $X_{[1]1n}$ | $X_{[2]1n}$ | $\cdots$ | $X_{[m]1n}$ |
| | 2 | $X_{[1]2n}$ | $X_{[2]2n}$ | $\cdots$ | $X_{[m]2n}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]mn}$ | $X_{[2]mn}$ | $\cdots$ | $X_{[m]mn}$ |

# RSS of Size $n_\bullet = nm \Longrightarrow n$ times a RSS of size $m$

| Cycle | Set | Ranks 1 | 2 | $\cdots$ | m |
|---|---|---|---|---|---|
| 1 | 1 | $X_{[1]11}$ | $X_{[2]11}$ | $\cdots$ | $X_{[m]11}$ |
| | 2 | $X_{[1]21}$ | $X_{[2]21}$ | $\cdots$ | $X_{[m]21}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m1}$ | $X_{[2]m1}$ | $\cdots$ | $X_{[m]m1}$ |
| 2 | 1 | $X_{[1]12}$ | $X_{[2]12}$ | $\cdots$ | $X_{[m]12}$ |
| | 2 | $X_{[1]22}$ | $X_{[2]22}$ | $\cdots$ | $X_{[m]22}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m2}$ | $X_{[2]m2}$ | $\cdots$ | $X_{[m]m2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| n | 1 | $X_{[1]1n}$ | $X_{[2]1n}$ | $\cdots$ | $X_{[m]1n}$ |
| | 2 | $X_{[1]2n}$ | $X_{[2]2n}$ | $\cdots$ | $X_{[m]2n}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]mn}$ | $X_{[2]mn}$ | $\cdots$ | $X_{[m]mn}$ |

# An economic version of RSS (VSR)

# Part 2

# Ranked Set Sampling (RSS)

Just a unit from each set?!

| | | Min | Second Min | $\cdots$ | Max |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $\cdots$ | $X_{[m]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $\cdots$ | $X_{[m]2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | m | $X_{[1]m}$ | $X_{[2]m}$ | $\cdots$ | $X_{[m]m}$ |

# Virtual Stratified Sampling Using RSS (VSR) Panahbehagh and Bruggemann (2017)

VSR of size $n_\bullet = \sum_{h=1}^{m} n_h$

- Select $K$ sets of size $m$ based on SRS
- Sort each set, based on an auxiliary variables to make $m$ strata
- Take a sample of size $n_h$ from stratum (rank) h.

| | | stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $\cdots$ | $X_{[m]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $\cdots$ | $X_{[m]2}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| | $K$ | $X_{[1]K}$ | $X_{[2]K}$ | $\cdots$ | $X_{[m]K}$ |

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- Example: VSR of size $n_\bullet = 12$, $m = 4$, $n_h = n_\bullet / m = 3$

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- $K > 3$ (Number of sets) for example $K = 5$

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- Example: VSR of size $n_\bullet = 12$, $m = 4$, $n_h = n_\bullet/m = 3$
- $K > 3$ (Number of sets) for example $K = 5$

| | | stratum 1 | stratum 2 | stratum 3 | stratum 4 |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $X_{[3]1}$ | $X_{[4]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $X_{[3]2}$ | $X_{[4]2}$ |
| | 3 | $X_{[1]3}$ | $X_{[2]3}$ | $X_{[3]3}$ | $X_{[4]3}$ |
| | 4 | $X_{[1]4}$ | $X_{[2]4}$ | $X_{[3]4}$ | $X_{[4]4}$ |
| | 5 | $X_{[1]5}$ | $X_{[2]5}$ | $X_{[3]5}$ | $X_{[4]5}$ |

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- Example: VSR of size $n_\bullet = 12$, $m = 4$, $n_h = n_\bullet / m = 3$
- $K > 3$ (Number of sets) for example $K = 5$

| | | stratum 1 | stratum 2 | stratum 3 | stratum 4 |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $X_{[3]1}$ | $X_{[4]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $X_{[3]2}$ | $X_{[4]2}$ |
| | 3 | $X_{[1]3}$ | $X_{[2]3}$ | $X_{[3]3}$ | $X_{[4]3}$ |
| | 4 | $X_{[1]4}$ | $X_{[2]4}$ | $X_{[3]4}$ | $X_{[4]4}$ |
| | 5 | $X_{[1]4}$ | $X_{[2]5}$ | $X_{[3]5}$ | $X_{[4]5}$ |

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- Example: VSR of size $n_\bullet = 12$, $m = 4$, $n_h = n_\bullet/m = 3$
- $K > 3$ (Number of sets) for example $K = 5$

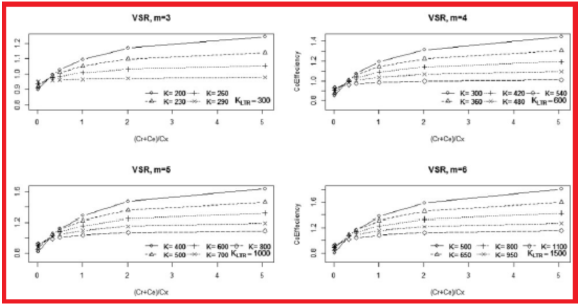
| | | stratum 1 | stratum 2 | stratum 3 | stratum 4 |
|---|---|---|---|---|---|
| | 1 | $X_{[1]1}$ | $X_{[2]1}$ | $X_{[3]1}$ | $X_{[4]1}$ |
| Sets | 2 | $X_{[1]2}$ | $X_{[2]2}$ | $X_{[3]2}$ | $X_{[4]2}$ |
| | 3 | $X_{[1]3}$ | $X_{[2]3}$ | $X_{[3]3}$ | $X_{[4]3}$ |
| | 4 | $X_{[1]4}$ | $X_{[2]4}$ | $X_{[3]4}$ | $X_{[4]4}$ |
| | 5 | $X_{[1]4}$ | $X_{[2]5}$ | $X_{[3]5}$ | $X_{[4]5}$ |

- For RSS with $n_\bullet = 12$ and $m = 4$ we need 12 sets!

# Virtual Stratified Sampling Using RSS (VSR)

With considering Cost and Precision simultaneously, VSR can be more efficient than RSS and SRS.

(Panahbehagh and Bruggemann (2017))



# Multivariate VSR

# Part 3

# Multivariate VSR

**Univariate $\Longrightarrow$ Multivariate**

# How to sort the data based on multivariate variable?!

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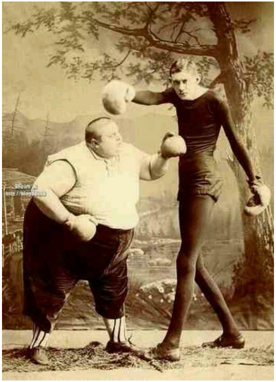
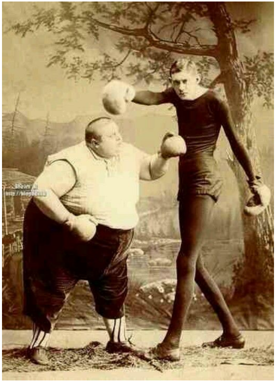
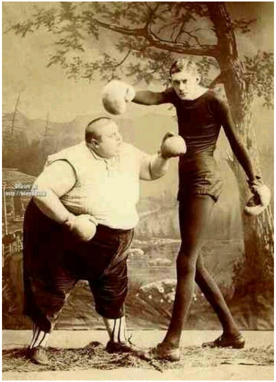


# How to sort the data based on multivariate variable?!

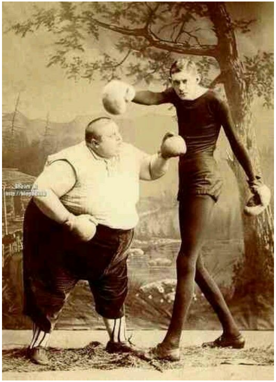


# How to sort the data based on multivariate variable?!



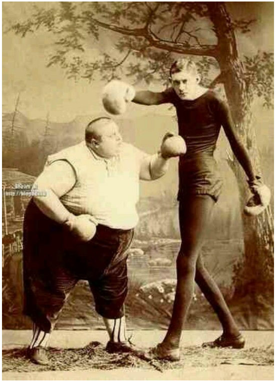
- Height? $\surd$

# How to sort the data based on multivariate variable?!



- Height? √
- Weight? √

# How to sort the data based on multivariate variable?!



- Height? √
- Weight? √
- Height and Weight? ?!?!

# How to sort the data based on multivariate variable?!

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1 Using just one of the variables (MVSR).

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2. Using a linear combination of the variables.

# How to sort the data based on multivariate variable?!

1. Using just one of the variables (MVSR).

2. Using a linear combination of the variables.

3. Using many initial sample.

# Multivariate VSR

- $\mathbf{X} \sim f_{\boldsymbol{\mu}}$, where $\mathbf{X} = (X^1, X^2, ..., X^R)$ ($1, 2, ..., R$ are indexes and not powers!)

$$E(\mathbf{X}) = \boldsymbol{\mu} = (\mu^1, \mu^2, ..., \mu^R).$$

- Main aim is to estimate $\boldsymbol{\mu}$.

- Our strategy is the same as VSR and just we sort the set units only based on $X^1$.

- Virtual strata, using conventional RSS, sorted based on $X^1$

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{[1]1}$ | $\mathbf{X}_{[2]1}$ | $\cdots$ | $\mathbf{X}_{[m]1}$ |
| $\mathbf{X}_{[1]2}$ | $\mathbf{X}_{[2]2}$ | $\cdots$ | $\mathbf{X}_{[m]2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\mathbf{X}_{[1]K}$ | $\mathbf{X}_{[2]K}$ | $\cdots$ | $\mathbf{X}_{[m]K}$ |

- Virtual strata, using conventional RSS, sorted based on $X^1$

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{[1]1}$ | $\mathbf{X}_{[2]1}$ | $\cdots$ | $\mathbf{X}_{[m]1}$ |
| $\mathbf{X}_{[1]2}$ | $\mathbf{X}_{[2]2}$ | $\cdots$ | $\mathbf{X}_{[m]2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\mathbf{X}_{[1]K}$ | $\mathbf{X}_{[2]K}$ | $\cdots$ | $\mathbf{X}_{[m]K}$ |

- Now we can take a sample like VSR, a SRS from each stratum.

- Virtual strata, using conventional RSS, sorted based on $X^1$

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{[1]1}$ | $\mathbf{X}_{[2]1}$ | $\cdots$ | $\mathbf{X}_{[m]1}$ |
| $\mathbf{X}_{[1]2}$ | $\mathbf{X}_{[2]2}$ | $\cdots$ | $\mathbf{X}_{[m]2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\mathbf{X}_{[1]K}$ | $\mathbf{X}_{[2]K}$ | $\cdots$ | $\mathbf{X}_{[m]K}$ |

- Now we can take a sample like VSR, a SRS from each stratum.

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{[1]1}$ | $\mathbf{X}_{[2]1}$ | $\cdots$ | $\mathbf{X}_{[m]1}$ |
| $\mathbf{X}_{[1]2}$ | $\mathbf{X}_{[2]2}$ | $\cdots$ | $\mathbf{X}_{[m]2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\mathbf{X}_{[1]K}$ | $\mathbf{X}_{[2]K}$ | $\cdots$ | $\mathbf{X}_{[m]K}$ |

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{[1]1}$ | $\mathbf{X}_{[2]1}$ | $\cdots$ | $\mathbf{X}_{[m]1}$ |
| $\mathbf{X}_{[1]2}$ | $\mathbf{X}_{[2]2}$ | $\cdots$ | $\mathbf{X}_{[m]2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\mathbf{X}_{[1]K}$ | $\mathbf{X}_{[2]K}$ | $\cdots$ | $\mathbf{X}_{[m]K}$ |

$$\widehat{\mu}_V^j = \frac{1}{m}\sum_{h=1}^{m} \bar{X}_{[h]}^j, \quad \bar{X}_{[h]}^j = \frac{1}{n}\sum_{i \epsilon s_h} X_{[h]i}^j$$

## Theorem

*In MVSR, $\widehat{\mu}_V^j$ is an unbiased estimator for $\mu^j$ and*

$$V(\widehat{\mu}_V^j) = \frac{1}{nm}(\sigma_j^2 - \frac{(1-\frac{n}{K})}{m}\sum_{h=1}^{m}(\mu_{[h]}^j - \mu^j)^2)$$

*and if $X_i^j = \mu^j + \rho_{1j}\frac{\sigma_j}{\sigma_1}(X_i^1 - \mu^1) + \varepsilon_i$ where $\varepsilon \perp X^1$, then*

$$V(\widehat{\mu}_V^j) = \frac{1}{nm}(\sigma_j^2 - \frac{(1-\frac{n}{K})}{m}\rho_{1j}^2\sum_{h=1}^{m}(\mu_{(h)}^j - \mu^j)^2)$$

*and at the end an unbiased estimator for $V(\widehat{\mu}_V^j)$ is*

$$\frac{K-1}{m(mK-1)}\sum_{h=1}^{m}\frac{1}{n(n-1)}\sum_{i\epsilon s_h}(X_{[h]i}^j - \bar{X}_{[h]}^j)^2 + \frac{1}{m(mK-1)}\sum_{h=1}^{m}(\bar{X}_{[h]}^j - \widehat{\mu}_V^j)^2$$

## Multivariate variable sampling based on Partial Order Theory

# Part 4

# Linear Extensions

Linear extensions are different projections of the partial order into a complete order that respect all the relations in the partial order set.

- An example with $R = 2$ and $m = 5$

| | $X^1$ | $X^2$ |
|---|---|---|
| a | 0 | 1 |
| b | 2 | 1 |
| c | 1 | 2 |
| d | 3 | 3 |
| e | 0 | 4 |

| LE1 | LE2 | LE3 | LE4 | LE5 | LE6 | LE7 | LE8 |
|---|---|---|---|---|---|---|---|
| d | d | d | e | d | d | d | e |
| c | c | e | d | b | b | e | d |
| b | e | c | c | c | e | b | b |
| e | b | b | b | e | c | c | c |
| a | a | a | a | a | a | a | a |

# Linear Extensions

- Two Designs to Rank Based on Partial Order Sets

I) **CPOR**; An example of **CPOR** with $R = 2$ and $m = 5$

| | $X^1$ | $X^2$ |
|---|---|---|
| a | 0 | 1 |
| b | 2 | 1 |
| c | 1 | 2 |
| d | 3 | 3 |
| e | 0 | 4 |

| LE1 | LE2 | LE3 | LE4 | LE5 | LE6 | LE7 | LE8 |
|---|---|---|---|---|---|---|---|
| d | d | d | e | d | d | d | e |
| c | c | e | d | b | b | e | d |
| b | e | c | c | c | e | b | b |
| e | b | b | b | e | c | c | c |
| a | a | a | a | a | a | a | a |

| | mean height | rounded height |
|---|---|---|
| a | 1 | 1 |
| b | 2.875 | 3 |
| c | 2.875 | 3 |
| d | 4.75 | 5 |
| e | 3.5 | 4 |

| strata | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | a | | b | e | d |
| | | | c | | |

# CPOR

I) **CPOR**;

| stratum 1 | stratum 2 | $\cdots$ | stratum m |
|---|---|---|---|
| $\mathbf{X}_{\{1\}1}$ | $\mathbf{X}_{\{2\}1}$ | $\cdots$ | $\mathbf{X}_{\{m\}1}$ |
| $\mathbf{X}_{\{1\}2}$ | $\mathbf{X}_{\{2\}2}$ | $\cdots$ | $\mathbf{X}_{\{m\}2}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\vdots$ | $\vdots$ | $\cdots$ | $\mathbf{X}_{\{m\}K_m}$ |
| $\mathbf{X}_{\{1\}K_1}$ | $\vdots$ | | |
| | $\mathbf{X}_{\{2\}K_2}$ | | |

# CPOR

Theorem

*In CPOR, $\widehat{\mu}_P^j$ is an unbiased estimator for $\mu^j$, where*

$$\widehat{\mu}_P^j = \sum_{h=1}^{m} W_h \bar{X}_{\{h\}}^j, \quad W_h = \frac{K_h}{Km}; \;\; \bar{X}_{\{h\}}^j = \frac{1}{n_h} \sum_{i \epsilon s_h} X_{\{h\}i}^j$$

$$Var(\widehat{\mu}_P^j)??!!$$

# Linear Extensions

- Two Designs to Rank Based on Partial Order Sets

II) **RPOR**; An example of **RPOR** with $R = 2$ and $m = 5$

| | $X^1$ | $X^2$ |
|---|---|---|
| a | 0 | 1 |
| b | 2 | 1 |
| c | 1 | 2 |
| d | 3 | 3 |
| e | 0 | 4 |

| LE1 | LE2 | LE3 | LE4 | LE5 | LE6 | LE7 | LE8 |
|---|---|---|---|---|---|---|---|
| d | d | d | e | d | d | d | e |
| c | c | e | d | b | b | e | d |
| b | e | c | c | c | e | b | b |
| e | b | b | b | e | c | c | c |
| a | a | a | a | a | a | a | a |

| strata | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | a | c | b | e | d |

# RPOR

Theorem

*In RPOR,* $\widehat{\mu}_R^j$ *is an unbiased estimator for* $\mu^j$ *where*

$$\widehat{\mu}_R^j = \frac{1}{m}\sum_{h=1}^{m}\bar{X}_{[h\}}^j; \quad \bar{X} = \frac{1}{n}\sum_{i\in s_h} X_{[h\}i}^j$$

*with variance*

$$V(\widehat{\mu}_R^j) = \frac{\sigma_j^2}{Km} + \frac{1}{m^2}\sum_{h=1}^{m}\frac{1-\frac{n}{K}}{n}E_M(\frac{1}{Q}\sum_{q=1}^{Q}S_{[h\}qjK}^2).$$

*where* $q = 1, 2, ..., Q$ *are all the possible combinations of LEs,and*

$$\widehat{V}(\widehat{\mu}_R^j) = \frac{1}{nm(Km-1)}[\sum_{h=1}^{m}\sum_{i\epsilon s_{[h\}}}(X_{[h\}i}^j - \widehat{\mu}_R^j)^2 + (K-n)\sum_{h=1}^{m}s_{[h\}j}^2].$$

# Simulations

**Part 5**

# How to compare designs?

To evaluate and compare the efficiency of the designs, we calculate

$$\text{Efficiency}(\widehat{\mu}_.) = \frac{V(\bar{y})}{\text{MSE}(\widehat{\mu}_.)}$$

where

- $\bar{y}$ is the sample mean of a simple random sample
- $\widehat{\mu}_.$ stands for
  - $\widehat{\mu}_V$ (MVSR design)
  - $\widehat{\mu}_C$ (CPOR design)
  - $\widehat{\mu}_R$ (RPOR design)
- MSE indicates mean square error

# Data for the simulations

- Bivariate Normal Distribution.
- Data of a Medical Flower; Chamomile (Panahbehagh and Bruggemann 2017).
- Data of Chemical Pollution.

# Bivariate Normal, $\mu^1 = \mu^2 = 0$, $\sigma_1 = \sigma_2 = 1$

| $m$ | $K$ | $n$ | $\rho$ | $\widehat{\mu}_V^1$ | $\widehat{\mu}_V^2$ | $\widehat{\mu}_C^1$ | $\widehat{\mu}_C^2$ | $\widehat{\mu}_R^1$ | $\widehat{\mu}_R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 12 | 4 | 0.3 | 1.49 | 1.00 | 1.16 | 1.12 | 1.12 | 1.11 |
| | | | 0.5 | 1.45 | 1.05 | 1.20 | 1.21 | 1.17 | 1.18 |
| | | | 0.7 | 1.47 | 1.17 | 1.27 | 1.30 | 1.26 | 1.26 |
| | | | 0.9 | 1.49 | 1.35 | 1.41 | 1.42 | 1.39 | 1.41 |
| | | 6 | 0.3 | 1.31 | 1.01 | 1.13 | 1.10 | 1.10 | 1.07 |
| | | | 0.5 | 1.30 | 1.05 | 1.16 | 1.14 | 1.13 | 1.11 |
| | | | 0.7 | 1.33 | 1.13 | 1.23 | 1.23 | 1.19 | 1.20 |
| | | | 0.9 | 1.32 | 1.23 | 1.31 | 1.31 | 1.29 | 1.27 |

# Data of a medical flower; Chamomile

$X^1$ : Flower dry weight (F)

$Y^1$ : Flower height; $\rho(X^1, Y^1) = 0.78$

$X^2$ : Essence (E)

$Y^2$ : Number of petals; $\rho(X^2, Y^2) = 0.71$

- Also $\rho(Y^1, Y^2) = 0.77$

# Data of a medical flower; Chamomile

| K | m | n | $\widehat{\mu}_V^1$(F) | $\widehat{\mu}_V^2$(E) | $\widehat{\mu}_C^1$(F) | $\widehat{\mu}_C^2$(E) | $\widehat{\mu}_R^1$(F) | $\widehat{\mu}_R^2$(E) |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 2 | 1.40 | 1.11 | 1.32 | 1.17 | 1.28 | 1.14 |
| | | 3 | 1.23 | 1.07 | 1.18 | 1.06 | 1.18 | 1.07 |
| | | 4 | 1.10 | 1.04 | 1.09 | 1.04 | 1.09 | 1.05 |
| | 5 | 2 | 1.63 | 1.18 | 1.45 | 1.24 | 1.45 | 1.23 |
| | | 3 | 1.35 | 1.10 | 1.26 | 1.11 | 1.24 | 1.13 |
| | | 4 | 1.14 | 1.05 | 1.11 | 1.06 | 1.11 | 1.06 |
| | 7 | 2 | 1.77 | 1.19 | 1.55 | 1.27 | 1.53 | 1.27 |
| | | 3 | 1.40 | 1.10 | 1.29 | 1.12 | 1.30 | 1.14 |
| | | 4 | 1.17 | 1.06 | 1.13 | 1.07 | 1.13 | 1.07 |
| 7 | 3 | 3 | 1.36 | 1.09 | 1.26 | 1.15 | 1.25 | 1.13 |
| | | 5 | 1.15 | 1.06 | 1.13 | 1.07 | 1.12 | 1.08 |
| | | 6 | 1.09 | 1.03 | 1.07 | 1.03 | 1.06 | 1.03 |
| | 5 | 3 | 1.58 | 1.16 | 1.43 | 1.22 | 1.40 | 1.20 |
| | | 5 | 1.23 | 1.07 | 1.18 | 1.08 | 1.17 | 1.09 |
| | | 6 | 1.10 | 1.03 | 1.08 | 1.04 | 1.08 | 1.04 |
| | 7 | 3 | 1.71 | 1.19 | 1.53 | 1.25 | 1.51 | 1.24 |
| | | 5 | 1.26 | 1.08 | 1.22 | 1.09 | 1.20 | 1.11 |
| | | 6 | 1.12 | 1.04 | 1.09 | 1.05 | 1.09 | 1.05 |
| | Ave. | | 1.32 | 1.09 | 1.24 | 1.12 | 1.23 | 1.12 |

# Chemical Pollution; Pb, Cd, Zn and S

- First Case; high correlation
  $X^1$ : Pb
  $X^2$ : Zn;
  $$\rho(X^1, X^2) = 0.60$$
- Second Case; low correlation
  $X^1$ : Cd
  $X^2$ : S;
  $$\rho(X^1, X^2) = 0.05$$

# $\rho(Pb, Zn) = 0.60$ and $\rho(Cd, S) = 0.05$

| m | K | n | $\widehat{\mu}_V^1$ Pb | $\widehat{\mu}_V^2$ Zn | $\widehat{\mu}_C^1$ Pb | $\widehat{\mu}_C^2$ Zn | $\widehat{\mu}_R^1$ Pb | $\widehat{\mu}_R^2$ Zn | $\widehat{\mu}_V^1$ Cd | $\widehat{\mu}_V^2$ S | $\widehat{\mu}_C^1$ Cd | $\widehat{\mu}_C^2$ S | $\widehat{\mu}_R^1$ Cd | $\widehat{\mu}_R^2$ S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 2 | 1.32 | 1.11 | 1.13 | 1.21 | 1.12 | 1.15 | 1.36 | 1.01 | 1.13 | 1.09 | 1.12 | 1.09 |
| | | 4 | 1.11 | 1.02 | 1.06 | 1.03 | 1.05 | 1.03 | 1.11 | 1.01 | 1.05 | 1.00 | 1.05 | 1.03 |
| | 7 | 2 | 1.41 | 1.11 | 1.15 | 1.16 | 1.10 | 1.12 | 1.41 | 0.99 | 1.16 | 1.08 | 1.11 | 1.07 |
| | | 4 | 1.25 | 1.08 | 1.11 | 1.10 | 1.09 | 1.10 | 1.16 | 1.01 | 1.00 | 1.01 | 1.03 | 1.06 |
| | 10 | 2 | 1.59 | 1.16 | 1.24 | 1.25 | 1.16 | 1.17 | 1.53 | 1.04 | 1.22 | 1.12 | 1.19 | 1.11 |
| | | 4 | 1.38 | 1.11 | 1.15 | 1.17 | 1.12 | 1.14 | 1.29 | 1.02 | 1.16 | 1.09 | 1.07 | 1.07 |
| 5 | 5 | 2 | 1.61 | 1.21 | 1.23 | 1.27 | 1.18 | 1.23 | 1.53 | 1.02 | 1.19 | 1.12 | 1.15 | 1.10 |
| | | 4 | 1.15 | 1.04 | 1.05 | 1.05 | 1.04 | 1.06 | 1.13 | 1.01 | 1.05 | 1.02 | 1.04 | 1.02 |
| | 7 | 2 | 1.69 | 1.21 | 1.23 | 1.30 | 1.21 | 1.26 | 1.62 | 1.00 | 1.23 | 1.11 | 1.18 | 1.08 |
| | | 4 | 1.35 | 1.14 | 1.14 | 1.15 | 1.13 | 1.16 | 1.33 | 1.00 | 1.12 | 1.04 | 1.09 | 1.04 |
| | 10 | 2 | 1.93 | 1.28 | 1.31 | 1.37 | 1.24 | 1.34 | 1.78 | 0.99 | 1.28 | 1.13 | 1.21 | 1.12 |
| | | 4 | 1.56 | 1.21 | 1.20 | 1.28 | 1.17 | 1.26 | 1.47 | 1.03 | 1.15 | 1.12 | 1.13 | 1.11 |
| 7 | 5 | 2 | 1.69 | 1.21 | 1.26 | 1.31 | 1.21 | 1.28 | 1.63 | 1.04 | 1.26 | 1.18 | 1.21 | 1.19 |
| | | 4 | 1.16 | 1.10 | 1.09 | 1.10 | 1.07 | 1.12 | 1.15 | 0.99 | 1.06 | 1.04 | 1.06 | 1.01 |
| | 7 | 2 | 1.90 | 1.28 | 1.27 | 1.35 | 1.29 | 1.32 | 1.85 | 1.03 | 1.30 | 1.12 | 1.28 | 1.09 |
| | | 4 | 1.45 | 1.19 | 1.19 | 1.19 | 1.17 | 1.20 | 1.37 | 1.02 | 1.17 | 1.07 | 1.17 | 1.07 |
| | 10 | 2 | 2.20 | 1.36 | 1.40 | 1.49 | 1.39 | 1.49 | 1.99 | 1.02 | 1.40 | 1.19 | 1.31 | 1.15 |
| | | 4 | 1.66 | 1.30 | 1.27 | 1.36 | 1.25 | 1.34 | 1.62 | 1.02 | 1.25 | 1.13 | 1.24 | 1.12 |
| | A. | | 1.52 | 1.17 | 1.19 | 1.23 | 1.17 | 1.21 | 1.46 | 1.01 | 1.18 | 1.09 | 1.15 | 1.09 |

# Why Ranking Based on Partial Order?

# Why Ranking Based on Partial Order?

- To consider all the variables simultaneously.

# Why Ranking Based on Partial Order?

- To consider all the variables simultaneously.
- To avoid ambiguity of weighting variables.

# Why Ranking Based on Partial Order?

- To consider all the variables simultaneously.
- To avoid ambiguity of weighting variables.
- To avoid needing any extra initial sample.

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